1 Capacitors and DC Circuits

1.1 Formulas

Definition of capacitance: $C = Q/V$

Energy stored in a capacitor:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Parallel plate capacitance w/o dielectric: $C = \epsilon_0 A/d$

Parallel plate capacitance w/ dielectric: $C = \kappa\epsilon_0 A/d$

Capacitors in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

Capacitors in parallel: $C = C_1 + C_2 + \cdots + C_n$

Definition of current: $i = \Delta Q/\Delta t$

Current and drift velocity: $I = nqv_d A$

Definition of resistance: $R = \Delta V/I$

Resistance of a wire: $R = \rho l/A$

Temperature variation of resistivity:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

Power dissipation in a resistor:

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

Steps: in application of Kirchhoff's Rules

- Label current: $i_1, i_2, i_3, \ldots$
- Node equation: $\sum i_{\text{in}} = \sum i_{\text{out}}$
- Loop equation: $\sum(\pm V) + \sum \mp iR = 0$

Resistors in series: $R = R_1 + R_2 + \cdots + R_n$

Resistors in parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

Charging in an RC circuit: $q(t) = Q(1 - e^{-t/RC})$

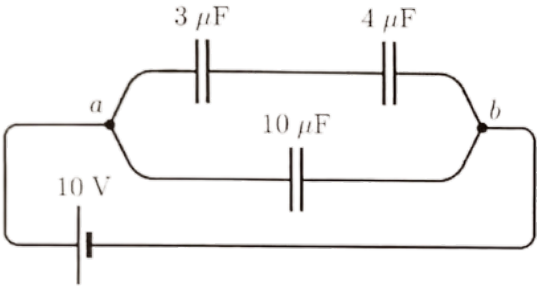
Discharging in an RC circuit: $q(t) = Qe^{-t/RC}$, where RC $= \tau$ is the time constant.

1.2 Electrons Through a Resistor Problem

If 5×10^{21} electrons pass through a 20Ω resistor in 10 min, what is the potential difference across the resistor? The fundamental charge is 1.602×10^{-19} C. Answer in units of V.

1.3 Voltage across a capacitor problem

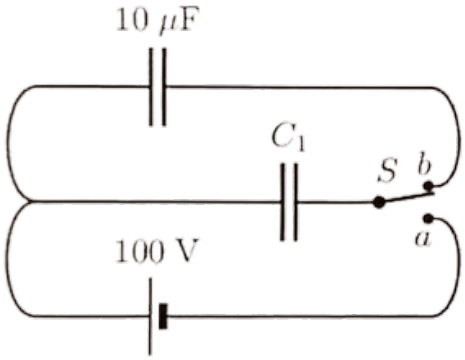
Consider the capacitor network



What is the voltage across the 4 μF (upper right hand) capacitor? Answer in units of V.

1.4 Unknown Capacitance Problem

When the switch is in position a, an isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the switch is moved to position b, this charged capacitor is then connected parallel to the uncharged 10 μF capacitor. The voltage across the combination becomes 30 V.



Calculate the unknown capacitance. Answer in units of μF.

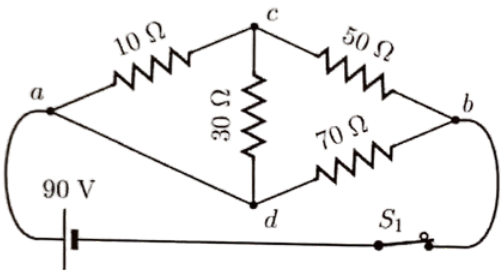
1.5 Drift Velocity Problem

An aluminum wire with a cross-sectional area of 4×10^{-6} m^2 carries a current of 5 A.

Find the drift speed of the electrons in the wire. Assume that each atom supplies one electron. Aluminum has a molecular weight of 26.98 g/mol and a density of 2.7 g/cm^3. Avogadro's number is 6.022×10^{23} and the fundamental charge is 1.602×10^{-19} C. Answer in units of m/s.

1.6 Equivalent Resistance Problem

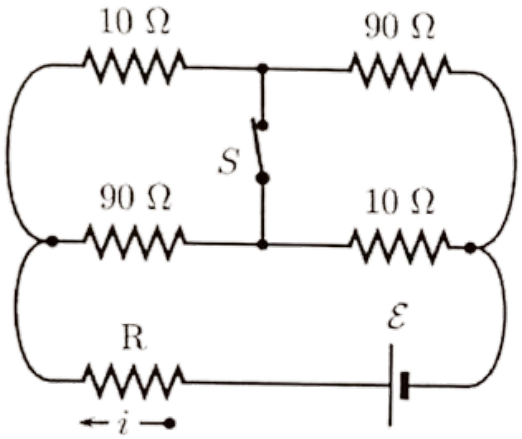
Four resistors are connected as shown in the figure.



Find the resistance between points a and b. Answer in units of Ω.

1.7 Find R in Circuit with Switch Problem

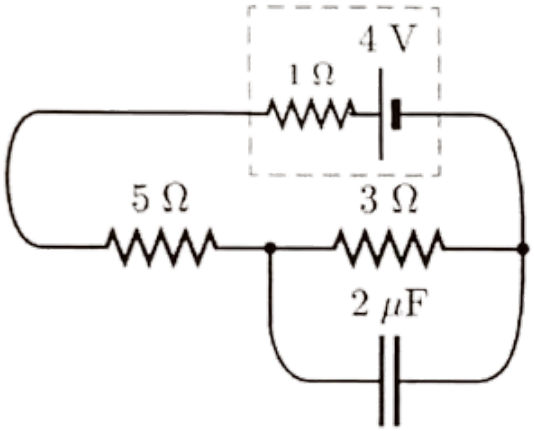
In the circuit shown below, the current i in the resistor R doubles its original value when the switch S is closed.



Find the value of R. Answer in units of Ω.

1.8 Charge on Capacitor Problem

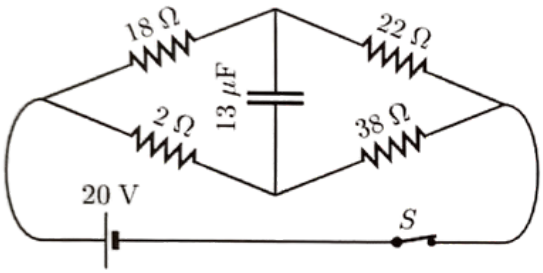
In the figure below the battery has an emf of 4 V and an internal resistance of 1Ω. Assume there is a steady current flowing in the circuit.



Find the charge on the 2 μF capacitor. Answer in units of μC.

1.9 Voltage across Capacitor Problem

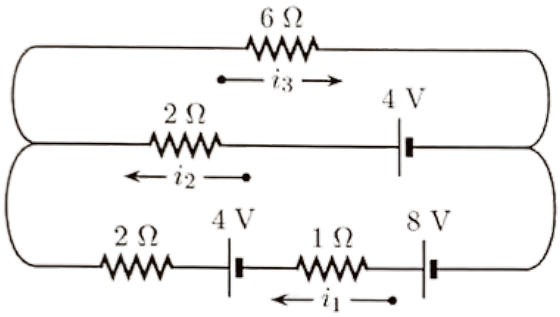
The circuit has been connected as shown in the figure for a "long" time.



What is the magnitude of the electric potential across the capacitor? Answer in units of V.

1.10 Two Loop Circuit Problem

Consider the circuit



Find i_1. Answer in units of A.